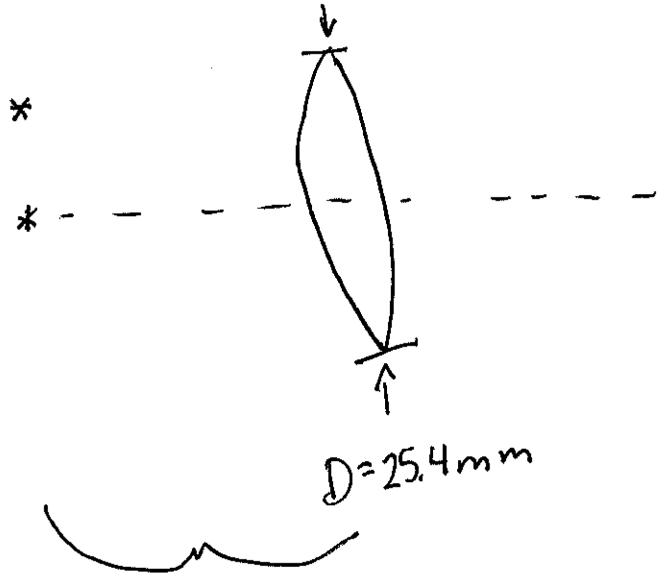
HW 11

(1)



Rayleigh Criterion: $\Delta\phi_{min} = 1.22\frac{\lambda}{D}$

(a) $\Delta y_{min} \approx f\,\Delta\phi_{min} = \frac{50.0\text{mm}\cdot 550\text{ nm}}{25.4\text{mm}}\cdot 1.22$

Dots must be at least $\Delta y_{min} = \boxed{1.32\ \mu m}$ apart

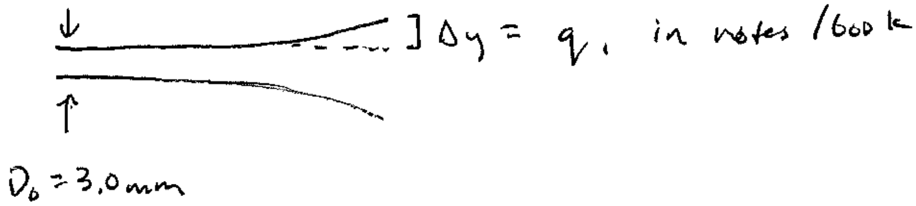
(b) $550\text{ nm} \doteq \frac{550\text{nm}\cdot 50.0\text{ mm}\cdot 1.22}{D}$

$\boxed{D \geq 61.0\text{ mm}}$

(c) $\Delta y_{min} = \frac{1.22 f}{D}\cdot\lambda = \frac{1.22 f}{D}\cdot\frac{\lambda_0}{n} = 1.22\,\frac{50.0\text{mm}}{61.0\text{mm}}\cdot\frac{550\text{nm}}{1.48}$

$\boxed{\text{resolution is better} \rightarrow 372\text{ nm}}$

(2)



$$D_{final} = D_0 + 2 \times \Delta y$$

$$\Delta y = 1.22 \frac{(100\,m)(632.8\,nm)}{3.0\,mm} = 25.7\ mm$$

$$D_{final} = 3.0\,mm + 51.4\,mm \approx \underline{54.4\ mm}$$

(3) (a-c) see next page

(d) $\Delta n = I n_2 = n \times 0.01$

(i) $$\frac{(1.55)(0.01)}{(2.19 \times 10^{-20}\,m^2/W)} = \underline{7.08 \times 10^{17}\ W/m^2}$$

(ii) $P = I \cdot A$

$$A = \left(\frac{D}{2}\right)^2 \cdot \pi = 1.29 \times 10^{-8}\ m^2$$

$$P = (7.08 \times 10^{17}\ W/m^2)(1.29 \times 10^{-8}\ m^2) = \underline{9.11 \times 10^{9}\ W}$$

$$\approx 9\ GW$$

Problem 3, HW11

(a) In a chirped pulse, the frequency of the carrier at the front of the pulse (arriving first) is either lower (up-chirp) or higher (down-chirp), than the frequency at the back of the pulse (arriving last).

(b) The Kerr effect causes an increase in the index of refraction that is highest in the center of the pulse. Thus, as the pulse propagates, the front half of the pulse is always experiencing and increasing N, which means it is always slowing down. This causes the wavefronts of the carrier frequency to separate (longer wavelengths = lower frequency, or “redder”). Alternatively, the back half of the pulse see a decreasing index and is speeding up. The wavefronts get close together, as the ones in back move faster than the ones in front of them (shorter wavelength = higher frequency, or “bluer”). Since the back of the pulse is higher in frequency, we call this an “upchirp”.

(c) In anomalous dispersion, the lower frequencies have higher n and move slower than higher frequencies. Thus, the components of a pulse (remember, a “pulse” means there is also a bandwidth, i.e. spread in frequencies) will spread out, with the “red” frequencies trailing behind the “bluer” frequencies. This is a down-chirp.

(4) $\tau_{pulse} = 100\ fs = 100 \times 10^{-15}\ s$

$\tau_{rep.} = 1/f_{rep.} \approx 11.8\ ns$

$\lambda = 800\ nm$

$\nu = c/\lambda = 375\ THz = 3.75 \times 10^{14}\ Hz$

$P = 0.5\ W\ (= P_{avg})$

(a) Energy per pulse = Energy per second · seconds per pulse

$E_{pulse} = P \cdot \tau_{rep} = \underline{5.90 \times 10^{-9}\ J}$

(b) $P_{peak} = P_{avg} \cdot \dfrac{\tau_{rep}}{\tau_{pulse}}$

$P_{peak} = \underline{59.0\ kW}$

(c) $\Delta\nu = \dfrac{1}{\tau_{pulse}} = 10\ THz = 1.0 \times 10^{13}\ Hz$

bandwidth in Hz

Range in frequency: $\underline{370\ THz - 380\ THz}$

($\rightarrow$ 10 THz, centered at 375 THz)

Range in wavelength: $\dfrac{c}{370\ THz}$ & $\dfrac{c}{380\ THz}$: $\underline{789\ nm - 810\ nm}$

(4 continued)

(d) $D_{beam} = 2.0$ mm

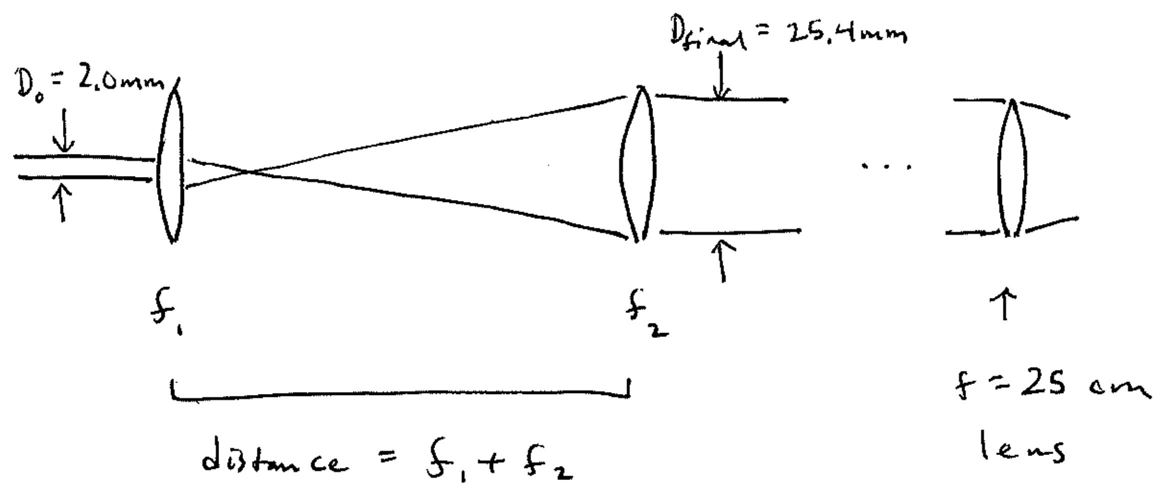
$f = 250$ mm

spot size = Airy disk diameter (~FWHM) $= \dfrac{1.22 f \lambda}{D} = 122\ \mu m$

$I_{peak} = \dfrac{P_{peak}}{A} = \dfrac{59.0\text{ kW}}{(61\,\mu m)^2 \pi} = \underline{5.05 \times 10^{12}\ W/m^2}$

$= 5.05 \times 10^{8}\ W/cm^2$

(e)



(i) $\dfrac{f_2}{f_2} = \dfrac{25.4\text{ mm}}{2.0\text{ mm}} \approx \underline{12.7}$ (or $\dfrac{f_1}{f_2} \approx 0.079$)

(ii) $D_{spot} = \dfrac{1.22 f \lambda}{D_{final}} = 9.61\ \mu m$

$I_{peak} = \dfrac{59.0\text{ kW}}{\left(\frac{9.6}{2}\right)^2 \pi} = \underline{8.13 \times 10^{14}\ W/m^2}$

(4 continued)

(e) (iii) $I = \frac{1}{2} c \varepsilon |E_0|^2$

$$E_{peak} = \sqrt{\frac{2 I_{peak}}{c \varepsilon}} = \underline{7.83 \times 10^8 \ \frac{V}{m}}$$

(f) (i) $r_{sun} = 696 \times 10^6 \ m$

$d_{sun\ to\ earth} = 147 \times 10^9 \ m$

$$\frac{I_{sun\ surface}}{I_{earth}} = \left(\frac{4\pi r_{sun}^2}{4\pi d_{sun\ to\ earth}^2} \right)^{-1}$$

$$I_{sun\ surface} = I_{earth} \times 44.6 \times 10^3$$

$$= \underline{6.25 \times 10^7 \ W/m^2}$$

<u>I_{peak} from pulsed laser is more!</u>

(ii) Air ionizes at ~ 3.0×10^6 V/m

E_{peak} from pulsed laser <u>will</u> ionize air.

Problem 5, HW11

In blackbody radiation, the relative intensity at each wavelength is set by the temperature. The exact form takes into account the Boltzmann distribution of particles at a given temperature. Typically, blackbody radiation is considered incoherent—by which, we mean that the coherence length is very short, even as short as the wavelength of light (in the case of sunlight).

In laser radiation, the length of the laser cavity (and the optical mode of the standing wave in the cavity) sets the central wavelength of the laser. The bandwidth of the laser is related to the Finesse of the laser cavity. Laser radiation is typically considered "coherent", meaning that the coherence length is macroscopic (can range from cm to km). However, some lasers are more coherent than others.

Extra Credit

1. 1 photon or fewer when $\langle n_{avg} \rangle = 1$;

$$P = P_{1\ photon} + P_{no\ photons}$$

$$P = \frac{1^1}{1!} e^{-1} + \frac{1^0}{0!} e^{-1} = 2e^{-1}$$

$$\underline{P \approx 73.6\%}$$

2. $P_{2\ photons} = \frac{1^2}{2!} e^{-1} = \frac{1}{2} e^{-1} \approx \underline{18.4\%}$

3. $P_{more\ than\ 1\ photon} = 1 - P_1 - P_0$

$$0.01 = 1 - \frac{\langle n_{avg} \rangle^1}{1!} e^{-\langle n_{avg} \rangle} - \frac{\langle n_{avg} \rangle^0}{0!} e^{-\langle n_{avg} \rangle}$$

$$0.99 = \frac{n_{avg} + 1}{e^{\langle n_{avg} \rangle}}$$

$$\underline{\langle n_{avg} \rangle \approx 0.15 \text{ photons}}$$